

Normal Distribution as Approximation to Binomial Distribution

Binomial Distribution has 4 requirements:

1. The experiment must have a fixed number of trials
2. The trials must be independent
3. Each trial must have all outcomes classified into two categories
4. The probabilities must remain constant for each trial.

We can find normal probabilities using the formula (tedious), a fancy calculator (still tedious), or the tables in the back of the book. While the tables are exact, they are limited. For example, what if $p=1/3$? Under certain conditions, we can approximate the binomial distribution using the normal distribution. The key here is approximate. This is not exact, but is usually close enough.

Normal Approximation to the Binomial distribution.

IF $np \geq 5$ AND $nq \geq 5$, then the binomial random variable is approximately normally distributed with mean $\mu = np$ and standard deviation $\sigma = \sqrt{npq}$.

PROBLEM! The binomial distribution is discrete, and the normal distribution is continuous. We must use a continuity correction (rounding in reverse). If x equals 35 in the binomial distribution, then x is between 34.5 and 35.5 in the normal distribution.

Procedure for Normal Approximation with Continuity Correction

Verify the binomial distribution applies

1. Try to see if the table works for this problem, it is exact. USE IT if possible
2. Verify that the assumptions are met - at least 5 successes and 5 failures
3. Calculate μ and σ
4. Draw your picture using x (don't worry about correction yet)
5. Decide which numbers round to x ($x - 0.5$ and $x + 0.5$)
6. If x is included in the shaded part, extend the shaded part to include all numbers that round to x
7. Determine the probabilities as before.

Example 1.

Find the exact and approximate probabilities

$n=14$, $p=0.40$, $P(8)$

Using the table in the back of the book, the exact probability is **0.092**

$$= n \cdot p = 14 \cdot 0.4 = 5.6$$

$$= \sqrt{npq} = \sqrt{14 \cdot 0.4 \cdot 0.6} = \sqrt{3.36} = 1.833$$

The conditions are met.

What numbers round to 8? 7.5 and 8.5.

Convert these to z-scores in the usual way

$$z = \frac{x - \mu}{\sigma} = \frac{7.5 - 5.6}{1.83303} = 1.04$$

$$z = \frac{x - \mu}{\sigma} = \frac{8.5 - 5.6}{1.83303} = 1.58$$

Look up the probabilities from the table and subtract

$$0.4429 - 0.3508 = \mathbf{0.0921}$$

Pretty good!

Example 2

Find the exact and approximate probabilities

$n=12$, $p=0.90$, $P(\text{fewer than } 10)$

Using the table in the back of the book, the exact probability is $.004+.021+.085 = \mathbf{.110}$

$$n \cdot p = 12 \cdot 0.9 = 10.8$$

$$n \cdot q = 12 \cdot 0.1 = 1.2$$

In this case the approximation is not suitable.

Example 3

A multiple choice tests consists of 25 questions with possible answers of a, b, c, d, and e.

Estimate the probability that with random guessing, the number of correct answers is between 3 and 10 inclusive.

$$= n \cdot p = 25 \cdot \frac{1}{5} = 5$$

$$= \sqrt{npq} = \sqrt{25 \cdot \frac{1}{5} \cdot \frac{4}{5}} = \sqrt{4} = 2$$

What numbers round to between 3 and 10 inclusive?

2.5 to 10.5 inclusive

Convert to z scores, etc.

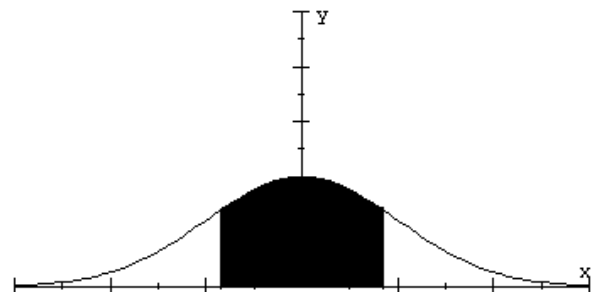
x corrected	2.5	10.5
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z	-1.25	2.75
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table	0.3944	0.497
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prob	0.8914	
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Why did I add the probabilities? Draw the picture



Example 4

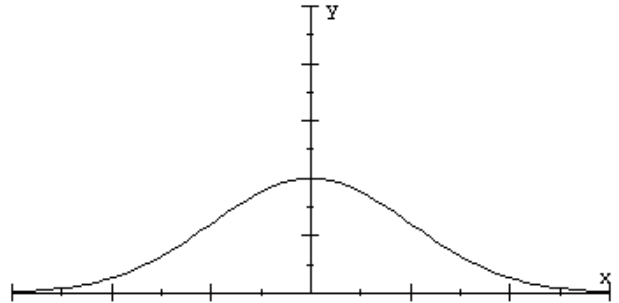
Nine percent of men are color blind. Researchers need at least 50 men with this trait, so they randomly select 600 men. Estimate the probability that at least 50 color blind men are in the sample

n 600
 p 0.09
 mu 54
 sigma 7.009993

At least 50 is 50 or more. That would be 49.5

and to the right

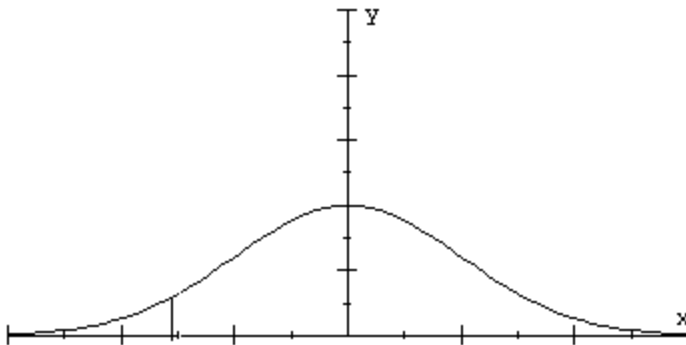
x corrected 49.5
 z -0.64
 table 0.2389
 prob 0.7389

**Example 5.**

20% of viewers watch 60 Minutes. Estimate the probability that fewer than 35 in a random sample of 200 households are watching 60 minutes.

n 200
 p 0.2
 mu 40
 sigma 5.656854

Fewer than 35 means less than 34. What number?

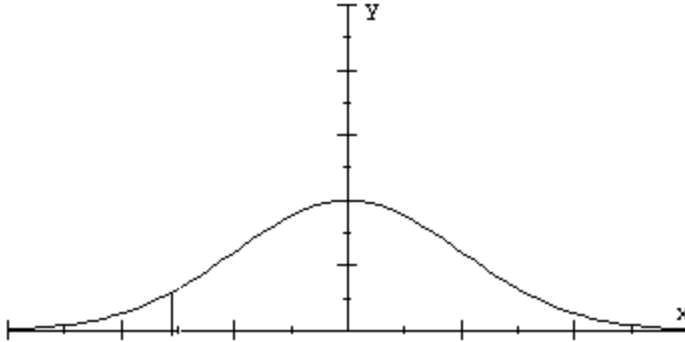


x corrected 34.5
 z -0.97
 table 0.3340
 prob 0.1660

Example 6

The probability of winning on a slot machine is 5% if person plays the machine 500 times, find the probability of winning 30 times.

$n = 500$ $p = 0.05$ $x = 30$ exactly
mean = 25 $sd = 4.873397$
between 29.5 and 30.5
 $z = 0.92$ and $z = 1.13$
table gives 0.3212 and 0.3708
Subtract to get the answer of 0.0496



Example 7

In a large university, 30% of the incoming freshmen elect to enroll in a Personal Finance course offered by the university. Find the probability that of 800 randomly selected incoming freshmen, at least 260 have elected to enroll in the course.

n 800 p 0.3 x at least 260, $x = 259.5$
mean 240 sd 12.96148
 z 1.50 table 0.4338 tail 0.0662

